**Pool Simulation - Impact Calculations for Collisions between Balls**

**Introduction:**

In the pool simulation, the velocities of two balls after impact are calculated using trigonometry and a combination of physics principles. Firstly, the system can be rotated about the center of one of the balls to make the axis of collision horizontal. After this rotation, the vertical component of the velocities becomes tangent to the axis of collision, meaning it will not change after the impact. Conservation of momentum and coefficient of restitution equations can then be applied to calculate the horizontal component of the velocities after impact. Lastly, we convert these velocities back to normal *x* and *y* axes to get the final velocities and angles of the balls.

**Initial Velocity and Angle:**

The initial velocities of the balls in the *x* and *y* directions are used to calculate the initial velocity and initial angle of the balls. The following equations can be used to show this step:

m/s

where is the initial velocity of the ball, is the initial velocity of the ball in the *x* direction, is the initial velocity of the ball in the *y* direction and is the initial angle from the positive *x* axis.

**Initial Velocities on Rotated Axis of Collision:**

Diagram

Description automatically generated

Figure 1: Diagram of the Axis of Collision.

The system is rotated around the center of the first ball to make the axis of collision horizontal. To do this, the angle is calculated using the following equation:

where is the vertical distance between the center of the balls, is the horizontal distance between the center of the balls and is the angle of collision from the positive *x* axis.

Now, the following equations are used to find the velocities of the balls in the *x* and *y* directions on the rotated axis of collision:

m/s

m/s

where is the initial velocity of the ball, is the initial velocity of the ball in the *x* direction after the axis is rotated, is the initial velocity of the ball in the *y* direction after the axis is rotated, is the initial angle from the positive *x* axis and is the angle of collision from the positive *x* axis.

Diagram, engineering drawing

Description automatically generated

Figure 2a: Diagram showing the velocities before rotation of the Axis of Collision.

Diagram

Description automatically generated

Figure 2b: Diagram showing the velocities after rotation of the Axis of Collision.

Notice that the velocities in *y* the direction are now tangent to the axis of collision meaning that they will not change after the collision.

**Final Velocities on Rotated Axis of Collision:**

The next step is to derive an equation to calculate the velocities of the balls in the *x* direction after the collision. The following conservation of momentum and coefficient of restitution equations are used:

where is the mass of the first ball, is the mass of the second ball, is the initial velocity of the first ball, is the initial velocity of the second ball, is the final velocity of the first ball, is the final velocity of the second ball and is the coefficient of restitution.

After rearranging the equations, we get:

m/s

m/s

Now, the second equation is substituted into the first and the variable is isolated. Since we are only using this equation for the velocities in the *x* direction, we can substitute those into the equation. These steps are shown below:

m/s

m/s

m/s

where is the final velocity of the first ball in the *x* direction after the axis is rotated, is the initial velocity of the first ball in the *x* direction before the axis is rotated, is the final velocity of the second ball in the *x* direction after the axis is rotated and is the initial velocity of the second ball in the *x* direction before the axis is rotated.

**Final Velocities on Normal Axis of Collision:**

The next step is to rotate the axis of collision back to its original form and get the velocities in the *x* and *y* directions in standard cartesian form. This is done using the following equations:

where is the final velocity of the ball in the *x* direction and is the final velocity of the ball in the *y* direction.

**Final Velocity and Angle:**

Lastly, we can use the final velocities in the *x* and *y* directions to calculate the final velocity and final angle of the balls. The following equations can be used to show this step:

m/s

where is the final velocity of the ball and is the final angle from the positive *x* axis.

**Conclusion:**

As shown in this document, the pool simulation uses trigonometry, conservation of momentum and coefficient of restitution equations to calculate the velocities of two balls after impact.